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CRITICAL DISTURBANCE THRESHOLDS OF WATER TRANSPORT SYSTEMS UNDER MULTIFACTOR UNCERTAINTY

Abstract

This study presents a mathematical formulation for determining critical disturbance thresholds of water transport configurations under multifactor uncertainty. Functional admissibility is defined by simultaneous constraints on delivery time, available capacity, operational risk, and reliability. Disturbances are represented by additional delay and relative changes in capacity, risk, and reliability. Analytical thresholds identify the disturbance levels at which individual constraints become active. An illustrative calculation yields a maximum additional delay of 12 h, a capacity reduction of 16.67%, a risk increase of 50%, and a reliability reduction of approximately 8.16%. A combined disturbance scenario satisfies all four requirements, whereas increasing capacity reduction to 18% lowers available capacity to 984 t against a required cargo volume of 1000 t. The formulation supports preliminary assessment of transport configurations and identification of exhausted functional reserves. Its scope is limited to a fixed configuration with separable disturbance effects; interactions between factors, rerouting, and recovery dynamics are not modelled. The numerical results illustrate the method and do not constitute operational standards.

Keywords: water transport systems; freight transportation; functional admissibility; functional resilience; multifactor uncertainty; critical disturbance thresholds; transport capacity; delivery time; operational risk; transport reliability.

1. Introduction

Water transport systems operate through interconnected shipping routes, ports, terminals, and inland transport links. The ability of such a system to perform a transport task depends on the availability and capacity of these elements, the duration of transport operations, and the reliability of individual links. Changes in any of these parameters can affect the conditions under which cargo delivery remains feasible.

Operational disturbances rarely affect only one system parameter. Port congestion may simultaneously increase waiting time and reduce terminal throughput; restrictions on a transport link may decrease available capacity and require route changes; adverse operating conditions may increase both transport time and risk. The resulting state of the transport system is therefore determined by a combination of disturbances rather than by the variation of a single parameter.

Existing transport optimization models provide methods for route selection, capacity allocation, scheduling, and decision-making under uncertain conditions [1-3]. Robust and stochastic approaches extend deterministic formulations by considering parameter variability and alternative disturbance scenarios [4-7]. Reliability and resilience studies, in turn, examine the ability of transport networks to maintain or restore their performance following disruptions [8-12]. These approaches provide the methodological basis for analysing transport systems exposed to changing operating conditions.

However, identifying an optimal or feasible solution under a specified scenario does not directly determine how far the system is from losing its required functionality. A transport configuration may remain feasible under several disturbance scenarios but become unacceptable after a relatively small additional change in travel time, available capacity, risk, or network availability. The

transition between these states is therefore of particular interest when the permissible operating range of a transport system has to be determined.

For this purpose, operational disturbances can be represented by a vector of simultaneously varying factors. The corresponding combinations of transport time, available capacity, operational risk, and reliability define a region in which the specified transport function can still be performed. The boundary of this region represents combinations of disturbance factors at which at least one functional requirement reaches its admissible limit. Such a representation makes it possible to move from the analysis of individual disturbance scenarios to the determination of critical disturbance thresholds.

The aim of this study is to develop a mathematical approach for determining critical disturbance thresholds in a water transport system under conditions of multifactorial uncertainty. The proposed formalized description interprets disturbances in terms of changes in transit time, throughput capacity, operational risk, and network availability, comparing them with specified functional requirements. The study focuses on identifying the region of admissible states, its boundaries, and the critical combinations of factors that lead to the loss of the required transport function.

The developed approach distinguishes three system states: operation within admissible limits, operation at the critical boundary, and violation of at least one functional requirement. Using a computational experiment, the study analyzes how these boundaries shift under various combinations of disturbances and identifies the factors that cause the system to transition from an acceptable state to a functionally unacceptable one.

2. Methods

2.1. Representation of the transport system under disturbances

The water transport system is represented by a directed network

$$G = (V, E), \tag{1}$$

where V is the set of transport and transshipment nodes and E is the set of transport links. A feasible transport configuration connects the origin node O with the destination node D and is characterized by transport time, available capacity, operational risk, and reliability.

Under nominal operating conditions, the state of a transport configuration is described by

$$\mathbf{Y}_0 = [T_0, C_0, R_0, Rel_0], \tag{2}$$

where T_0 is the nominal transport time, C_0 is the available transport capacity, R_0 is the operational risk, and Rel_0 is the reliability of the transport configuration.

Operational disturbances are represented by the vector

$$\boldsymbol{\xi} = [\xi_T, \xi_C, \xi_R, \xi_A], \tag{3}$$

where ξ_T represents additional transport delay, ξ_C represents capacity reduction, ξ_R represents an increase in operational risk, and ξ_A represents reduced availability of network elements.

The disturbed state is therefore expressed as

$$\mathbf{Y}(\boldsymbol{\xi}) = [T(\boldsymbol{\xi}), C(\boldsymbol{\xi}), R(\boldsymbol{\xi}), Rel(\boldsymbol{\xi})]. \tag{4}$$

For the computational formulation, the individual state variables are defined as

$$T(\xi) = T_0 + \xi_T, \tag{5}$$

$$C(\xi) = C_0(1-\xi_C), \qquad 0 \le \xi_C \le 1, \tag{6}$$

$$R(\xi) = R_0(1+\xi_R), \qquad \xi_R \ge 0, \tag{7}$$

and

$$Rel(\xi) = Rel_0(1-\xi_A), \qquad 0 \le \xi_A \le 1. \tag{8}$$

Thus, ξ_T is expressed in units of time, whereas ξ_C, ξ_R, and ξ_A describe relative changes in the corresponding system parameters.

2.2. Functional requirements

A transport configuration is considered functionally admissible if, under the disturbance vector ξ, it satisfies the delivery-time constraint $T(\xi) \le T_{\max}$, the capacity constraint $C(\xi) \ge Q$, the operational-risk constraint $R(\xi) \le R_{\max}$, and the reliability constraint $Rel(\xi) \ge Rel_{\min}$, where $T_{\max}$ is the maximum permissible delivery time, Q is the required cargo volume, Rmax is the maximum admissible operational risk, and Relmin is the minimum acceptable reliability level. These constraints define the admissible disturbance region $\Omega_F = \{\xi : T(\xi) \le T_{\max}, C(\xi) \ge Q, R(\xi) \le R_{\max}, Rel(\xi) \ge Rel_{\min}\}$, thus $\xi \in \Omega_F$ indicates that all specified transport requirements are satisfied, whereas $\xi \notin \Omega_F$ indicates violation of at least one requirement, rendering the configuration functionally inadmissible under the corresponding disturbance conditions.

2.3. Critical disturbance thresholds

A critical disturbance state $F\xi^{\text{crit}} \in \partial\Omega_F$ lies on the boundary of the admissible functional region, where at least one functional constraint reaches its limiting value while the remaining constraints remain satisfied. The corresponding critical conditions are $T(\xi^{\text{crit}}) = T_{\max}, C(\xi^{\text{crit}}) = Q, R(\xi^{\text{crit}}) = R_{\max}, or\ Rel(\xi^{\text{crit}}) = Rel_{\min}$. Individual disturbance thresholds are determined by substituting the response functions from Eqs. (5)–(8) into these equalities and solving for the relevant disturbance component, with the other components held fixed. The delivery-time threshold therefore represents the disturbance level at which the available temporal reserve is exhausted.

For capacity reduction, $\xi_C^{crit} = 1 - \dfrac{Q}{C_0}$ provided that $C_0 \geq Q$.

For increasing operational risk, $\xi_R^{crit} = \dfrac{R_{\max}}{R_0} - 1$, and for decreasing availability/reliability, $\xi_A^{crit} = 1 - \dfrac{Rel_{\min}}{Rel_0}$.

These quantities represent the maximum disturbance magnitude that can be absorbed by the considered transport configuration before the corresponding functional requirement is violated.

3. Results

An illustrative calculation was performed for a fixed transport configuration with

$(T_0 = 48)h, (T_{\max} = 60)h, (C_0 = 1200)t, (Q = 1000)t, (R_0 = 0.04), (R_{\max} = 0.06), (Rel_0 = 0.98),$ and $(Rel_{\min} = 0.90)$.

Capacity and cargo demand refer to the same transport task; risk is represented by a dimensionless index on a common scale. All parameters are

assumed for demonstration purposes. The resulting thresholds are presented in Table 1.

Table 1. Illustrative critical disturbance thresholds.

Disturbance	Threshold calculation	Critical value
Additional delivery time	$(T_{\max} - T_0)$	12 h
Relative capacity reduction	$(1 - Q / C_0)$	0.1667
Relative risk increase	$(R_{\max} / R_0 - 1)$	0.5000
Relative reliability reduction	$(1 - Rel_{\min} / Rel_0)$	0.0816

For the combined disturbance vector $\xi = (8\text{h}, 0.10, 0.25, 0.05)$, the resulting delivery time, capacity, risk, and reliability are 56 h, 1080 t, 0.05, and 0.931, respectively. All requirements remain satisfied. Increasing only the capacity reduction to 0.18 decreases available capacity to 984 t, making the configuration inadmissible despite compliance with the other requirements. At a capacity reduction of (1/6), capacity equals the required 1000 t, placing the configuration on the admissibility boundary.

4. Discussion

The computational experiment demonstrates how the calculated disturbance thresholds distinguish between normal, marginal, and critical operating modes. The adopted additive response model evaluates adjacent disturbances without accounting for their inter-element interactions. Accordingly, meeting one criterion does not offset a violation of another. The model operates with a static configuration and does not perform real-time re-routing of cargo flows or estimate recovery time lags. Implementation of the method requires calibrating parameters to a given

topology and adapting reliability estimates, whereas the calculations presented here are purely illustrative.

5. Conclusions

The proposed approach formalizes the functional feasibility of the system through aggregate constraints on delivery time, capacity, risk, and reliability, identifying the critical points at which these constraints are triggered. The results show that exceeding the limits of even a single parameter renders the entire configuration inoperable, even with a high margin of safety for the remaining indicators. The method is effective for the initial screening of transportation schemes and the identification of depleted resources. A direction for further research is to account for the cumulative effects of disturbances and to calculate the network's recovery capacity through alternative routes.

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